Synopsis of the Ph.D. Thesis entitled

A STUDY ON DOMINATING SETS

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Synopsis

A STUDY ON DOMINATING SETS

Graph Theory is an important branch of mathematics. It has grown rapidly in recent times with a lot of research activities because of its applications in diverse fields which include Computer science (Algorithms and computation), Biochemistry (Genomics), Electrical Engineering (Communication networks and coding theory) and Operations research. One of the main emerging concepts in Graph theory is Domination in graphs. Domination arises in facility location problem and has a variety of applications in fields such as Linear Algebra and Optimization, Online Social Networks, Cloud Architecture for Video Distribution Services, Computer Communication Networks and Wireless Sensor Networks.

Let $G = (V, E)$ be a simple graph. A set $D \subseteq V$ is a **dominating set** of G if every vertex in $V - D$ is adjacent to a vertex in D. The **domination number** of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set.

A thorough study of domination appears in [9, 10]. Many domination parameters are introduced by imposing additional constraints on the dominating set D or on the dominated set $V - D$ or on the method by which vertices in $V - D$ are dominated.

In our thesis we deal with two domination parameters : Restrained domination and Antipodal domination. Restrained domination was already introduced by Domke et al.[5] in 1999, whereas the antipodal domination is a new parameter introduced by us.

Restrained domination is defined by imposing a condition on the dominated set. A set $S \subseteq V$ is a **restrained dominating set** if every vertex not in S is adjacent to a vertex in S and to a vertex in V - S. The **restrained domination number** of G, denoted by $\gamma_r(G)$, is the smallest cardinality of a restrained dominating set of G. By the definition $\gamma(G) \leq \gamma_r(G)$.

An application of the concept of restrained domination is that of prisoners and guards[5]. Each vertex not in the restrained dominating set corresponds to a position of a prisoner, and every vertex in the restrained dominating set corresponds to a position of a guard. Note that each prisoner's position is observed by a guard's position (for effective security) while each prisoner's position is seen from at least one other prisoner's position (to protect the rights of prisoners). To be cost effective, it is desirable to place as few guards as possible.

We have introduced Antipodal domination by imposing a condition on the dominating set. A dominating set $S \subseteq V$ is said to be an **Antipodal Dominating Set(ADS)** of a connected graph G if there exist vertices $x, y \in S$ such that $d(x, y) = diam(G)$. The minimum cardinality of an ADS is called the **Antipodal Domination Number(ADN)**, and is denoted by $\gamma_{ap}(G)$.

Any country is under threat and faces a lot of challenges from internal and external threats, which calls for a robust defence mechanism. To manage external security threats, border forces are required to monitor the international borders against intrusion. To manage internal security threats, internal security forces are needed. Besides law and order, their main duties are rescue and relief operations at the time of natural calamities, participating in UN peace keeping mission, etc. To minimize the cost and maximize the benefits, the border forces are to be deployed at the farthest places and any country needs at least two border forces; moreover, the number of internal security forces is to be minimized but at the same time, they must be deployed at close range of unsecured places.

We can model this situation using a graph with the vertices representing the regions of the country and two vertices are adjacent if the corresponding regions are nearer. Now the problem of minimizing the number of border forces and internal security forces is equivalent to finding an antipodal dominating set with the minimum cardinality.

In our thesis, we have also introduced a new polynomial on graphs. Graph polynomials, provide a powerful tool in the area of graphical enumeration. As such, it encodes information about the graph, and enables algebraic methods for extracting this information. The first graph polynomial studied in literature is the chromatic polynomial, which counts the number of proper colorings of graphs (a coloring of the vertices such that adjacent vertices do not have the same color). It was defined by G.D. Birkhoff [3] to attack the famous four color problem in 1912.

In the literature, there are many domination related polynomials such as domination polynomial[1], total domination polynomial, connected domination polynomial, independent domination polynomial, the bipartition polynomial [15], pendant domination polynomial [17], Global bipartite domination polynomial[13] and Edge connected domination polynomial[11].

In our thesis, we introduce and initiate the study of restrained domination polynomial of graphs.

Summary of our Results

We present our results in **five chapters**. Chapters 2 and 3 deal with antipodal domination, Chapters 4 and 5 deal with restrained domination and Chapter 6 deals with restrained domination polynomial.

All our graphs are simple and undirected. Throughout our thesis, n and m denote the order and the size of the graph respectively.

Chapter 2 : Antipodal Domination in Graphs

In this chapter, we introduce a new domination parameter called antipodal domination by imposing condition on the dominating set S.

Definition. A dominating set $S \subseteq V$ is said to be an **Antipodal Dominating Set(ADS)** of a connected graph G, if there exist vertices $x, y \in S$ such that $d(x, y) = diam(G)$. The minimum cardinality of an ADS is called the **Antipodal Domination Number(ADN)**, and is denoted by $\gamma_{ap}(G)$.

We can easily extend this definition for disconnected graphs as follows :

Let G be a disconnected graph with the components $G_1, G_2, ..., G_k$. A set S is said to be an ADS of G if S can be written as $S = \bigcup_{i=1}^{k} S_i$, where each S_i is an ADS of G_i. Now $\gamma_{ap}(G) = \sum_{i=1}^{k} \gamma_{ap}(G_i)$.

Our first result deals with the relation between γ and γ_{ap}.

Theorem 1. *For any graph G with $k(\geq 1)$ components,*
$\gamma(G) \leq \gamma_{ap}(G) \leq \gamma(G) + 2k$ *and these bounds are sharp.*

Next we determine the ADN for standard graphs such as complete graphs, paths, cycles, complete bipartite graphs, wheels, generalized wheels, double stars and wounded spiders.

Also, we determine ADN of Jahangir graphs.

For $t \geq 3$, a **Jahangir graph**[2] $J_{s,t}$ is the graph on $st + 1$ vertices, consisting of a cycle $C_{st} : u_1u_2...u_{st}u_1$, with an additional vertex v that is adjacent to t vertices $u_s, u_{2s}, ..., u_{ts}$. Note that $V(J_{s,t}) = \{u_1, u_2, ..., u_{st}, v\}$ and
$E(J_{s,t}) = \{vu_{js} \mid 1 \leq j \leq t\} \cup \{u_iu_{i+1} \mid 1 \leq i \leq st - 1\} \cup \{u_{st}u_1\}$.

Theorem 2. $\gamma_{ap}(J_{2,t}) = \begin{cases} \lceil \frac{t}{2} \rceil & \text{if } t = 3 \\ \lceil \frac{t}{2} \rceil + 1 & \text{if } t \in \{5, 6, 7, 9\}. \end{cases}$

Theorem 3. $\gamma_{ap}(J_{2,t}) = \lceil \frac{t}{2} \rceil + 2$ *if* $t \geq 10$ *or* $t = 8$.

[These results are published in the Proceedings of the National Conference on Recent Developments on Emerging Fields in Pure and Applied Mathematics(2015), 65-72, ISBN No. : 978-93-83209-02-6.]

Moreover we study antipodal domination for operations on graphs.

Theorem 4. *For any two graphs* G_1 *and* G_2, $\gamma_{ap}(G_1 + G_2) \leq 3$. *Strict inequality holds if one of the following holds:*
(i) Both G_1 *and* G_2 *are complete*
(ii) $i(G_1) = 2$ *or* $i(G_2) = 2$ *(where* $i(G)$ *denotes the independent domination number of* G*).*

Theorem 5. *For any non-trivial graph* H *of order* n, $\gamma_{ap}(H \circ K_1) = n$.

Theorem 6. *For* $2 \leq s \leq t$, $\gamma_{ap}(K_s \times K_t) = s$.

Theorem 7. $\gamma_{ap}(K_s[K_t]) = \gamma_{ap}(K_s \boxtimes K_t) = \begin{cases} 1 & \text{if } s = t = 1 \\ 2 & \text{otherwise.} \end{cases}$

Theorem 8. $\gamma_{ap}(K_s \otimes K_t) = \begin{cases} 3 & \text{if } min(s,t) \geq 3 \\ st & \text{if } min(s,t) = 1 \text{ or } s = t = 2 \\ 2 & \text{otherwise.} \end{cases}$

Theorem 9. *For any* $k \geq 3$, $\gamma_{ap}(P_2 \times P_k) = \begin{cases} \frac{k+3}{2} & \text{if } k \equiv 1 (mod\ 4) \\ \lceil \frac{k+1}{2} \rceil & \text{otherwise.} \end{cases}$

Theorem 10. *For* $k \geq 2$, $\gamma_{ap}(P_2 \boxtimes P_k) = \lceil \frac{k+2}{3} \rceil$.

Theorem 11. *For* $k \geq 2$, $\gamma_{ap}(P_2[P_k]) = \begin{cases} 2 & \text{if } k \leq 6 \\ 3 & \text{otherwise.} \end{cases}$

Theorem 12. *For* $k \geq 2$, $\gamma_{ap}(P_2 \otimes P_k) = 2(\lceil \frac{k-1}{3} \rceil + 1)$.

Chapter 3 : Bounds of Antipodal Domination

In this chapter we derive bounds for γ_{ap}.

First we derive a bound in terms of order.

Theorem 1. For any non-trivial connected graph G of order n, $\gamma_{ap}(G) \leq \frac{n}{2} + 1$.

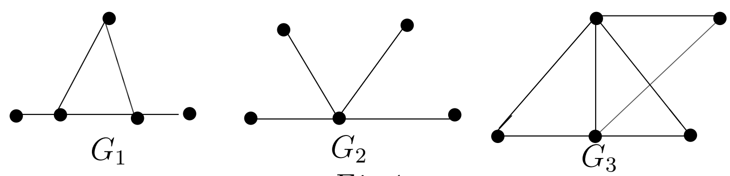
Using the above bound, we characterize the connected graphs with $\gamma_{ap}(G) = n, n-1$ and $n-2$.

Theorem 2. Let G be a connected graph of order n. Then

(i) $\gamma_{ap}(G) = n$ iff G is K_2 or K_1.

(ii) $\gamma_{ap}(G) = n-1$ iff G is K_3, P_3 or $K_{1,3}$.

Theorem 3. Let $G = (V, E)$ be a connected graph of order n. Then $\gamma_{ap}(G) = n-2$ iff $G \in \mathcal{A}$, where $\mathcal{A} = \{K_4, K_4-e, K_{1,3}+e, C_4, P_4, P_5, D_{1,2}, D_{2,2}, K_{1,4}, G_1, G_2, G_3\}$(Refer Fig.1 for G_1, G_2, G_3).



$Fig.1$

Next, we derive a bound for trees and characterize the trees that attain this bound.

Theorem 4. Let T be a non-trivial tree of order n with l pendant vertices. Then $\gamma_{ap}(T) \leq n - l + 2$.

Theorem 5. Let T be a tree of order $n \geq 3$, with l pendant vertices. Then $\gamma_{ap}(T) = n - l + 2$ iff

(i) Every vertex of T is either a pendant vertex or a support vertex and

(ii) For every pair of vertices x and y with $d(x, y) = diam(T), d(z) \geq 3$ for every $z \in N(x) \cup N(y)$.

[Theorems 1 -5 are published in Mathematical Sciences International Research Journal:Volume 5, Issue 2 (2016), 27-30, ISSN 2278-8697, ISBN 978-93-84124-93-9]

Next, we study bounds in terms of maximum degree, minimum degree and order.

Theorem 6. Let G be a connected graph with $\Delta(G) \leq n-2$. Then $\gamma_{ap}(G) \leq n - \Delta(G) + 1$. Moreover this bound is sharp.

Theorem 7. Let G be any non-trivial graph with $\Delta(G) = n-1$. Then $\gamma_{ap}(G) \leq 3$. Equality holds iff $\beta(G) \geq 3$, where $\beta(G)$ denotes the independence number of G.

Theorem 8. Let G be a graph with $diam(G) = 2$. Then $\gamma_{ap}(G) \leq \delta(G) + 2$. Furthermore equality holds iff $\delta(G) = 1$ and $d(w) \leq n-3$, for all $w \in V - N[v]$, where v is a vertex of minimum degree.

Next, we study graph complements and derive a Nordhaus-Gaddum type bound.

Theorem 9. If $diam(G) \geq 3$, then $\gamma_{ap}(\overline{G}) \leq 4$.

Theorem 10. If G is disconnected graph, then $\gamma_{ap}(\overline{G}) \leq 3$.

Theorem 11. If G is a connected graph of order n, then
$4 \leq \gamma_{ap}(G) + \gamma_{ap}(\overline{G}) \leq n + 2$.

Chapter 4 : Restrained domination number of graphs

In the literature, bounds of γ_r are determined in terms of order[8], maximum degree[4] and minimum degree[8] for graphs with $\delta(G) \geq 2$. We have derived bound in terms of clique number $\omega(G)$ and this bound can be applied for graphs with $\delta(G) = 1$ also.

Theorem 1. For any graph G with $\omega(G) \geq 3$, $\gamma_r(G) \leq n - \omega(G) + 1$. Moreover equality holds iff $G \in \mathcal{G}$ or $G = G_1 \cup G_2$, where $\mathcal{G}$ is the collection of graphs obtained by adding zero or more leaves to at most $k-1$ vertices of $K_k, k \geq 3$, $G_1 \in \mathcal{G}$ and G_2 is a galaxy.

Next, we deal with graph complements. Nirmala Vasantha [16] has proved that $\gamma_r(\overline{G}) \leq 3$, for connected graphs G with at least two pendant vertices. We deal with the graphs having exactly one pendant vertex.

Theorem 2. Let G be a connected graph with exactly one pendant vertex and $\Delta(G) \leq n-3$. Then $\gamma_r(\overline{G}) = 2$.

Theorem 3. Let G be a graph of order $n \geq 4$ having exactly one pendant vertex and $\Delta(G) = n-1$. Then

(i) $2 \leq \gamma_r(\overline{G}) \leq n$.

(ii) $\gamma_r(\overline{G}) = 2$ iff $G \cong K_1 + (K_1 \cup H)$, where H is a graph with $\Delta(H) \leq n-4$.

(iii) $\gamma_r(\overline{G}) = n$ iff G is the graph obtained from K_{n-1}, by attaching a pendant vertex to exactly one vertex of K_{n-1}.

Theorem 4. If G is a connected graph of order $n \geq 5$ having exactly one pendant vertex and $\Delta(G) = n-2$, then $2 \leq \gamma_r(\overline{G}) \leq n-2$. Both these bounds are sharp.

Theorem 5. Let G be a connected graph of order $n \geq 5$, having exactly one pendant vertex and $\Delta(G) = n-2$. Then $\gamma_r(\overline{G}) = n-2$ iff $\overline{G}$ is a graph obtained from P_4 by attaching one or more leaves to exactly one stem of P_4 or it can be obtained from C_3 by attaching exactly one pendant vertex to a vertex of C_3 and two or more pendant vertices to another vertex of C_3.

Next, we deal with some characterizations. In the literature, the characterization of graphs with $\gamma_r(G) = n, n-2, n-3, \Delta$ are studied in [5, 18].

We have characterized the trees with $\gamma_r(T) = n-4, \Delta+1$. These characterizations hold for some families of trees. We use the following notation for these families of trees.

Notation

In all the following families, we use a phrase 'attaching a stem', which means attaching a stem with one or more leaves.

1. $\mathcal{F}_1$ denotes the family of trees obtained from P_9 or P_8 by attaching zero or more leaves to the center(s) or the support vertices of the path.

2. $\mathcal{F}_2$ denotes the family of trees obtained from P_5, P_7, by attaching two stems to the center and zero or more leaves to the support vertices of the path; and the trees obtained from P_6, by attaching zero or more leaves to the support vertices and by either attaching two stems to one of the center or attaching one stem to each of the centers.

3. $\mathcal{F}_3$ denotes the family of trees obtained from P_5, P_6, P_7, by attaching one stem to exactly one center, one or more leaves to exactly one center, and zero or more leaves to the support vertices of the path.

4. $\mathcal{F}_4$ denotes the family of trees obtained from P_7 by attaching exactly one stem and zero leaves to the center, and zero or more leaves to all the other internal vertices of the path such that at least one neighbour of the center is of degree 2.

5. $\mathcal{F}_5$ denotes the family of trees obtained from P_7 by attaching zero or

more leaves to all the internal vertices of the path such that the center or at least one neighbour of the center (in the path) is of degree 2.

6. $\mathcal{F}_6$ denotes the family of trees obtained from P_6 by attaching one or more leaves to both the centers, and zero or more leaves to the support vertices of the path.

7. $\mathcal{T}_1$ denotes the family of trees obtained from wounded spiders with $\Delta \geq 3$, by attaching exactly one leaf to a vertex of degree two.

8. $\mathcal{T}_2$ denotes the family of trees obtained from wounded spiders with $\Delta \geq 3$, by attaching exactly one stem to a vertex of degree two, where the new attached stem has exactly one leaf.

9. $\mathcal{T}_3$ denotes the family of trees obtained from stars by subdividing each edge zero time or twice.

10. $\mathcal{T}_4$ denotes the family of trees obtained from stars by subdividing exactly one edge thrice, one edge once and the remaining edges zero time or once.

11. $\mathcal{T}_5$ denotes the family of trees obtained from stars by subdividing each edge exactly once.

12. $\mathcal{T}_6$ denotes the family of trees obtained from stars by subdividing exactly one edge four times, one edge zero time and the remaining edges zero time or once.

13. $\mathcal{T}_7$ denotes the family of trees obtained from stars by subdividing exactly one edge twice, one edge zero time and the remaining edges zero time or once.

Theorem 6. If T is a tree, $\gamma_r(T) = n - 4$ iff $T \in \bigcup_{i=1}^{5} \mathcal{F}_i$.

Theorem 7. For any tree T with $n \geq 4$, $\gamma_r(T) = \Delta + 1$ iff $T \in \bigcup_{i=1}^{7} \mathcal{T}_i$.

[Theorems 6 and 7 are published in International Journal of Applied Engineering Research, Volume 14, Number 3 (2019), ISSN 0973-4562.]

Chapter 5 : Restrained domination number of Jump graphs

In [6, 7], the jump distances and jump graphs $J_k(G)$, $(1 \leq k \leq m)$ are defined. $J_1(G)$ is referred as the **jump graph** of G and it is the graph whose vertices are the edges of G and two vertices of J(G) are adjacent iff the corresponding edges of G are non-adjacent, and it is denoted by $J(G)$. Note that $J(G)$ is defined only for non-empty graphs.

It is clear that, if G is a graph of size m, then $\gamma_r(J(G)) \leq m$.

In this chapter, first we determine the RDN for jump graph of standard graphs such as paths, cycles, complete bipartite graphs, stars, double stars and fans.

The first general result deals with trees.

Theorem 1. Let T be a tree.

(i) If $diam(T) = 4$, then $2 \leq \gamma_r(J(T)) \leq 3$.

(ii) If $diam(T) \geq 5$, then $\gamma_r(J(T)) = 2$.

Note that stars and double stars are the trees of diameter less than or equal to 3 and are dealt already.

Next, we have derived results on the structural properties of jump graphs. Using these properties, we characterize the graphs with $\gamma_r(J(G)) = 1, m$.

Theorem 2. $\gamma_r(J(G)) = 1$ iff $G \cong K_2 \cup G_1$, where G_1 is a graph in which each edge of G_1 is not adjacent to at least one edge of G_1.

Theorem 3. $\gamma_r(J(G)) = m$ iff

$$G \in \{K_2 \cup K_3, K_2 \cup K_{1,k}, K_3, K_{1,k}, D_{k,1}, K_1 + (K_2 \cup \overline{K_k})\}.$$

The next two results deal with $\gamma_r(G)$ and $\gamma_r(J(G))$.

Theorem 4. (i) $3 \leq \gamma_r(G) + \gamma_r(J(G)) \leq n + m$.

(ii) $2 \leq \gamma_r(G).\gamma_r(J(G)) \leq nm$.

Furthermore these bounds are best possible.

Theorem 5. $\gamma_r(G) + \gamma_r(J(G)) = n + m$ iff $G = K_{1,n-1}$ or $K_2 \cup K_{1,n-3}$.

Next, we derive a bound in terms of edge independence number and characterize the graphs that attain this bound.

Theorem 6. Let G be a connected graph with $\beta_1(G) \geq 3$. Then $\gamma_r(J(G)) \leq m - \beta_1(G) + 1$, where $\beta_1(G)$ is the edge independence number of G. Moreover equality holds iff $J(G) \in \mathcal{G}^*$, where $\mathcal{G}^* = \mathcal{C} \cup \{K_k \mid k \geq 4\}$ and $\mathcal{C}$ is the family of graphs obtained from K_3 by adding at most two leaves to at most two vertices of K_3.

Next, we derive a Nordhaus-Gaddum type bound.

Theorem 7. Let G be a non-empty graph. Then

(i) $2 \leq \gamma_r(J(G)) + \gamma_r(\overline{J(G)}) \leq m + 2$, except for the graphs $G \ncong P_3 \cup K_2$ and $G \ncong P_4$. Furthermore these bounds are best possible.

(ii) $\gamma_r(J(G)) + \gamma_r(\overline{J(G)}) = 2m$, when $G \cong P_3 \cup K_2$ or $G \cong P_4$.

Chapter 6 : Restrained domination polynomials

In this chapter, we introduce restrained domination polynomial of graphs.

Definition. Let G be a graph of order n and size m. Let $d_r(G, i)$ be the number of restrained dominating sets with cardinality i. Then the **restrained domination polynomial(RDP)** of G, denoted by $D_r(G, x)$ is defined as $D_r(G, x) = \sum\limits_{i=\gamma_r(G)}^{n} d_r(G, i)x^i$.

Theorem 1. If a graph G consists of k components $G_1, G_2, ..., G_k (k \geq 2)$, then $D_r(G, x) = \prod\limits_{i=1}^{k} D_r(G_i, x)$.

Theorem 2. For all $n_1, n_2 \geq 2$,

$D_r(K_{n_1,n_2}, x) = x^{n_1+n_2} + [(1 + x)^{n_1} - (1 + x^{n_1})][(1 + x)^{n_2} - (1 + x^{n_2})]$.

Theorem 3. $D_r(K_n \circ K_1, x) = x^n[(1 + x)^n] - nx^{2n-1}$.

Next, we have determined $d_r(G, i)$ for paths, cycles and the product of graphs; and using these results, we determine the RDP for these graphs.

Theorem 4. For $n \geq 6$, $D_r(P_n, x) = x^2[D_r(P_{n-2}, x) + 2D_r(P_{n-4}, x) + D_r(P_{n-6}, x)]$, where we set $D_r(P_0, x) = 0$.

Theorem 5. $D_r(C_n, x) = 3D_r(P_{n-2}, x) + D_r(P_n, x)$.

Theorem 6. For $k \geq 3$,

$$\begin{aligned} D_r(K_2 \times K_k, x) = & \sum_{t=2}^{k-1}[\sum_{t_1=\lfloor \frac{t}{2} \rfloor+1}^{t-1} {}_kC_{t_1}\ {}_kC_{t-t_1}]2x^t + \sum_{t=k}^{2k-4}[\ {}_kC_{t-k} + k\ {}_{(k-1)}C_{t-k+1} \\ & + \sum_{t_1=\lfloor \frac{t}{2} \rfloor+1}^{k-2} {}_kC_{t_1}\ {}_kC_{t-t_1}]2x^t + \sum_{\substack{t=2 \\ t\ even}}^{2k-4} ({}_kC_{\frac{t}{2}})^2x^t \\ & + [\ {}_kC_{k-3} + k\ {}_{(k-1)}C_{k-2}]2x^{2k-3} + k^2x^{2k-2} + x^{2k}. \end{aligned}$$

Theorem 7. For $k \geq 3$,

$$\begin{aligned} D_r(K_2 \otimes K_k, x) = & kx^2 + \sum_{t=3}^{k-1}[(t-1)\ {}_kC_{t-1} + \sum_{t_1=\lfloor \frac{t}{2} \rfloor+1}^{t-2} {}_kC_{t_1}\ {}_kC_{t-t_1}]2x^t + \sum_{t=k}^{2k-4}[k\ {}_{k-1}C_{t-k} \\ & + \sum_{t_1=\lfloor \frac{t}{2} \rfloor+1}^{k-2} {}_kC_{t_1}\ {}_kC_{t-t_1}]2x^t + \sum_{\substack{t=4 \\ t\ even}}^{2k-4} ({}_kC_{\frac{t}{2}})^2x^t + 2k\ {}_{k-1}C_{k-3}x^{2k-3} \\ & + k(k-1)x^{2k-2} + x^{2k}. \end{aligned}$$

Theorem 8. $D_r(K_2[K_k]) = D_r(K_2 \boxtimes K_k) = (1 + x)^{2k} - (1 + 2kx^{2k-1})$.

Conclusion

In our thesis, we have studied antipodal domination, restrained domination and restrained domination polynomial.

In Chapter 2, we have introduced a new type of domination called antipodal domination by imposing a condition on the dominating set and have determined the antipodal domination number for standard graphs, Jahangir graphs, and graphs that are obtained from various graph operations.

In Chapter 3, we have derived bounds for antipodal domination number in terms of order, maximum degree and minimum degree. Also we have characterized the graphs with $\gamma_{ap}(G) = n, n-1, n-2$. Next we have derived a Nordhaus-Gaddum type bound.

In Chapter 4, we have derived a bound for the restrained domination in terms of clique number. Next we have determined the RDN for graph complement. Moreover we have characterized the trees with $\gamma_r(T) = n-4, \Delta+1$.

In Chapter 5, we have determined the restrained domination number for Jump Graphs of standard graphs and trees. We have also derived the structural properties of jump graphs, and using these structural properties, we have characterized the graphs with $\gamma_r(J(G)) = 1, m$. Moreover we have derived a bound for RDN of jump graphs.

In Chapter 6, we have introduced restrained domination polynomial and have determined the restrained domination polynomial for paths, cycles and various products of K_2 with K_k.

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